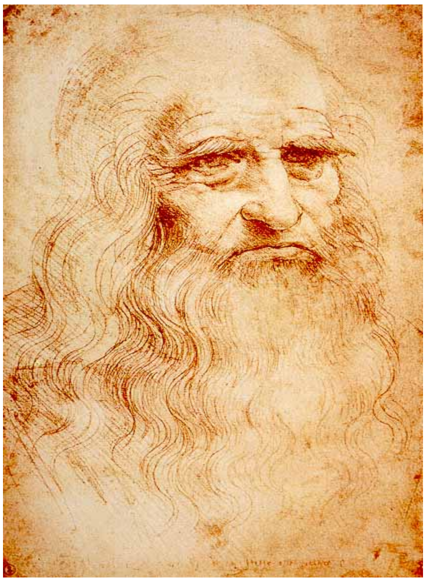
Leonardo Da Vinci's Self Supporting Bridge

John Burwell



Leonardo Da Vinci was an artist, inventor and engineer in the 15th century. His vision and calibre of mind allowed him to comprehend the twin worlds of science and art; he has often been described as the archetype of the "Renaissance man", a man whose seemingly infinite curiosity was equalled only by his powers of invention [1]. He was a master of innovation, ingenuity and engineering.

.

He invented the self supporting bridge sometime between 1485-1487 to be used as an emergency bridge for troops in times of war [2]. This unique design is held together by its own weight without requiring any ties or connections, in fact when a downward force is applied to the structure the braced members are forced to interlock and tighten together through the structural concepts of shear and bending. Da Vinci called it "The Bridge of Safety" [3].

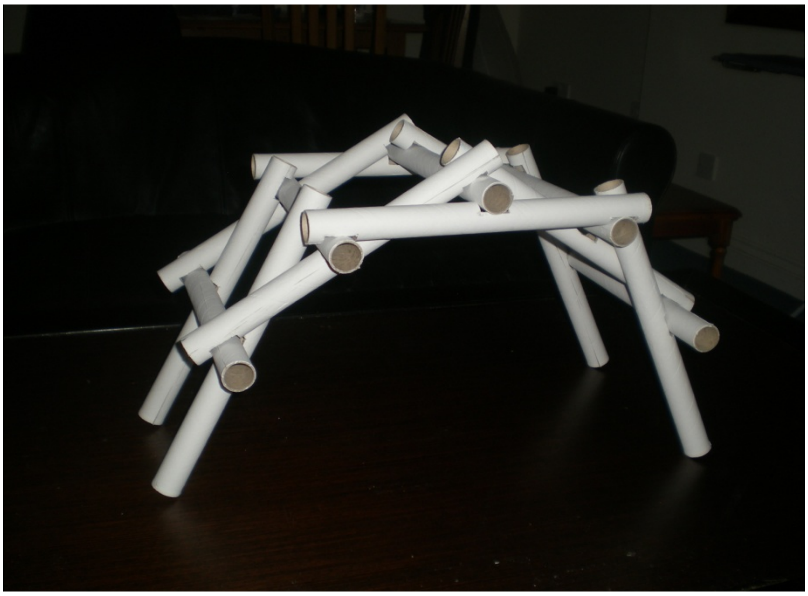


Figure 1

This replica model is made from cardboard tubes 30cm long. When the bridge is assembled and no additional weight is applied the pins are slightly loose and can slid back and forth due to the low self weight of the cardboard tubes.

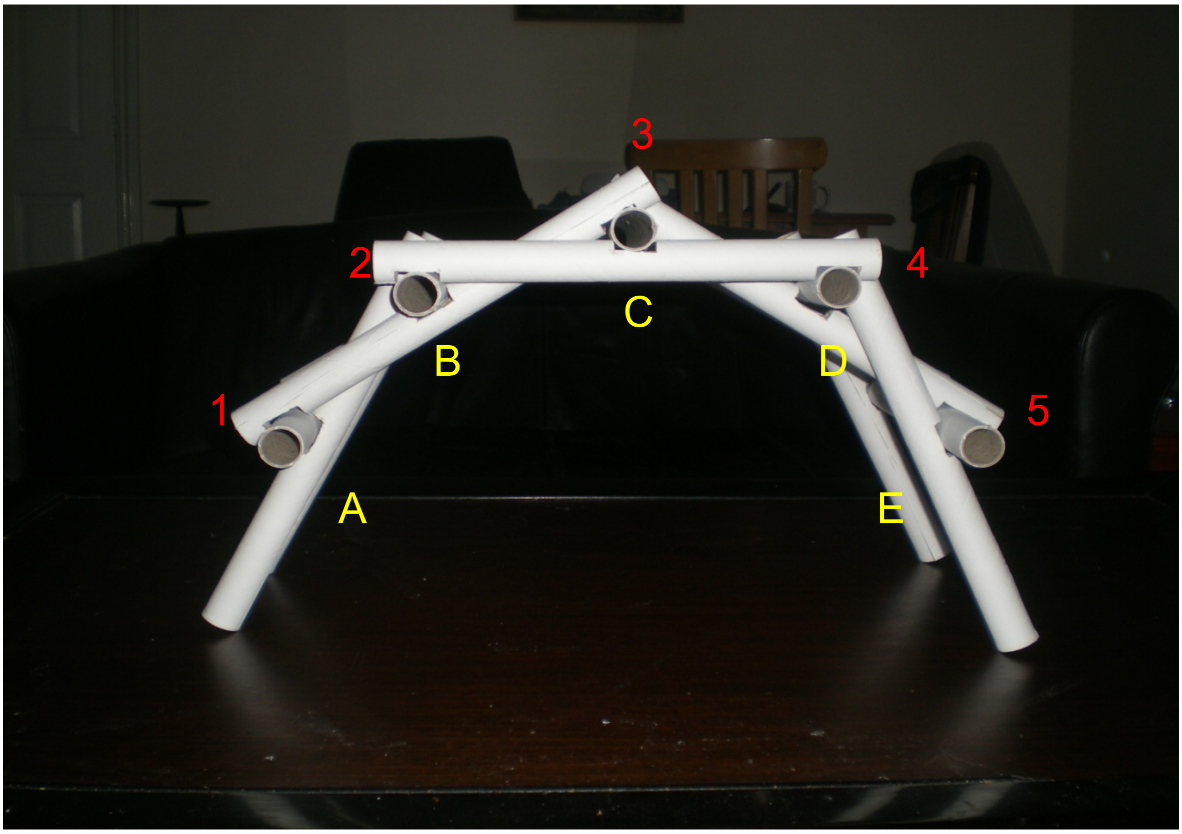
Figure 2

A downward load of 24.5N (2.5Kg * 9.81) is applied to the central pin and is spread throughout the structure which then becomes rigid and the pins are no longer loose. The shear force exerted by the load bends the beams, forcing them to make the beam/pin connections tighten, therefore increasing the stability of the bridge.

Figure 3

However, it would be incorrect to state that the strength of the bridge increases linearly with the size of the applied load. After a certain point the weight will have a negative effect. In figure 3 the load was doubled to 49N, the bridge passed its yield point and the members started deform. This picture was taken just before failure through plastic hinges forming on the underside of the horizontal beams below the central pin.

Analysis



Although the bridge is a simple design, its 2D statically analysis is quite complex. If a vertical downward load of W is applied to the central pin 3, what is the downward force exerted on the horizontal beam C? Here are two different solutions to the problem, which one do you agree with? (Or neither?)

Analysis 1

For load W on pin 3 reactions at each foot of bridge must be W/2;
thinking only of static equilibrium for each beam in vertical direction;
force on pin 2 from A is W/2; force on A from B through pin 1 is W;
force on pin 3 from B must also be W and by symmetry from D also W;
force on pin3 is 2W from Band D and with load on pin 3 of W, load on C from 3 is 3W;
that makes force on pin 2 from C to be 3W/2;
force of pin 2 on B is 3W/2+W/2=2W which is consistent with reactions at the ends of B of W.

Analysis 2

Let E_5 be the force exerted by pin 5 on beam E.
So $-E_5$ is the force exerted on pin 5 by beam E, and hence on beam D by pin 5.
Following analysis1, the force exerted by the ground on beam E is $\frac{1}{2}W$ vertically upwards. Considering the moments on beam E about pin 4 we have:

$$2 \times \tfrac{1}{2} W \cos\beta \quad = \quad E_5 \sin\varphi \qquad \text{(taking the length of the beam to be 2 units)}$$

where β is the angle between beam E and the horizontal and φ is the angle between beam E and the force E_5.

Considering the moments on beam D about pin 4 we have:

$E_5 \sin \varphi = D_3$

φ is the angle between beam D and the force D_5. This is so because of the isosceles triangle formed by the two tangents to pin 5.

The downward force on beam C is therefore:

$C_3 = W + 2 D_3 \cos \alpha$

where α is the angle between beam D and the horizontal [4].

Sources

[1] http://www.leonardo-da-vinci-biography.com/ accessed 18/2/2010
[2] http://www.ancientengineering.com/ accessed 18/2/2010
[3] Gibbs-Smith, C., 1978. The Inventions of Leonardo Da Vinci. Oxford: Phaidon Press Ltd.
[4] http://www.settledistrictu3a.org.uk/U3A-MATHS/dh/U3A/Maths/bridge.html accessed 20/2/2010

Images

http://picasaweb.google.com/lh/photo/sj5M5l9Zg9Lx4N-1Q-Zpog
http://www.davincibio.org/
Figures 1-4 my own work.